

Some Brief Comments on “Martingale-valued  
Measures,  
Ornstein-Uhlenbeck Processes with Jumps  
and Operator Self-Decomposability in Hilbert  
Space”

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It is stated on lines 13-14 of page 187 that the stochastic convolution process  $X_{J,C}$  inherits strongly càdlàg paths from the Lévy process  $X$ . It is not known if this is correct. There is a nice discussion of this on pages 155-63 of S. Peszat and J. Zabczyk, *Stochastic Partial Differential Equations with Lévy noise. An Evolution Equation Approach.*, Cambridge University Press, (2007). For example if the semigroup  $(S(t), t \geq 0)$  consists of generalised contractions, i.e.  $\|S(t)\| \leq e^{ct}$  for all  $t \geq 0$  where  $c \geq 0$  and the Lévy process  $X$  is a square-integrable martingale then  $X_{J,C}$  has strongly càdlàg paths (see Theorem 9.20 therein). A counter-example where the stochastic convolution fails to be strongly càdlàg is presented in Proposition 9.25 of that book but this is outside of our context as the semigroup and noise live on different Hilbert spaces (one of which is embedded in the other) and the extension of the semigroup to the larger space is no longer strongly continuous.

Note that in this paper, we only use the hypothetical strongly càdlàg property to prove convergence in distribution in Theorem 8. This still holds without the càdlàg assumption since the stronger convergence in probability is true here - see Lemma 2.1 in D.Applebaum *Potential Analysis*, **26**.