

Author's comment on "Some  $L^2$  Properties of semigroups of measures on Lie groups."

The definition of Lévy measure given in this paper isn't quite right. It is defined to be a Borel measure on  $G - \{e\}$  for which

$$\int_{G-\{e\}} \left( \sum_{i=1}^n x_i(\sigma)^2 \wedge 1 \right) \nu(d\sigma) < \infty,$$

where  $x_i \in C_c^\infty(G)$  ( $1 \leq i \leq n$ ) and  $(x_1, \dots, x_n)$  are canonical co-ordinates in a neighbourhood of  $e$ . This is similar to the definition that works in  $\mathbb{R}^d$  but the problem here is that it tells us nothing about  $\nu$  outside the union of the supports of the  $x_i$ s. The correct definition (as in M. Liao "Lévy Processes in Lie Groups", p.12) is to require

$$\int_{G-\{e\}} \left( \sum_{i=1}^n x_i(\sigma)^2 \right) \nu(d\sigma) < \infty \text{ and } \nu(U^c) < \infty$$

for all neighbourhoods  $U$  of  $e$ .

Also for Theorem 3.1,  $G$  needs to be a *compact* group equipped with normalised Haar measure. Otherwise, on line 3 of the proof, we will not have  $k_t \in L^2(G \times G)$ .